Algebra 1 – Things to remember

Exponents

$x^0 = 1$

$x^{-m} = \frac{1}{x^m}$

$x^m \cdot x^n = x^{m+n}$

$(x^m)^n = x^{mn}$

$\frac{x^m}{x^n} = x^{m-n}$

$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$

$(xy)^n = x^n y^n$

Absolute Value Equations and Inequalities.

$|x| = a \rightarrow \quad x = a \text{ or } x = -a$

$|x| < a \rightarrow \quad -a < x < a$

$|x| > a \rightarrow \quad x > a \text{ or } x < -a$

Examples of Special Cases.

$|x-6| + 7 > 2$
-7 -7

$|x-6| > -5$ All Real Numbers

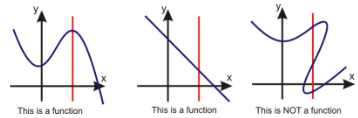
$|x+2| = -4$ No Solution

$|x+12| \leq -1$ No Solution

Functions

A **relation** is a set of ordered pairs.
A **function** is a relation in which every domain value is paired with exactly one range value.

Vertical line test: used to determine whether a relation is a function. If any vertical line crosses the graph of a relation more than once, the relation is not a function.



Domain: x-values used.
Range: y-values used.

Sequences

Arithmetic

Recursive: $a_n = a_{n-1} + d, a_1 = \#$

Explicit: $a_n = a_1 + (n-1)d$

Geometric:

Recursive: $a_n = r \cdot a_{n-1}, \ a_1 = \#$

Explicit: $a_n = a_1 \cdot r^{n-1}$

Factoring: look to see if there is a **GCF** (greatest common factor) **first**.
ab + ac = a(b +c)

Sum and Difference of Squares:
$a^2 + b^2$ = PRIME
$a^2 - b^2 = (a + b)(a - b)$

Perfect Square Trinomials

$a^2 + 2ab + b^2 = (a+b)^2$

$a^2 - 2ab + b^2 = (a-b)^2$

Ex: $x^2 - 6x + 9 = (x-3)^2$
$4x^2 + 20x + 25 = (2x+5)^2$

Simple Quadratic Trinomial (a=1)
$x^2 - 10x + 16 = (x-2)(x-8)$

Complex Quad. Trinomial (a≠1)
$2x^2 - 7x - 15 = (x-5)(2x+3)$

Factoring by Grouping
$x^3 + 2x^2 - 3x - 6 = (x^2-3)(x+2)$
$(x^3 + 2x^2) - (3x+6)$ group
$x^2(x+2) - 3(x+2)$ factor each
$(x^2-3)(x+2)$ factor

Linear Functions

Standard Form: $Ax + By = C$
Slope-Intercept Form: $y = mx + b$
Point-Slope: $y - y_1 = m(x - x_1)$
Slope: $\frac{y_2 - y_1}{x_x - x_1} = m = \frac{rise}{run}$
Rate of Change = $\frac{\text{change in dependent variable}}{\text{change in independent variable}}$

Linear Functions have a *constant* rate of change.
Horizontal Lines (y=k) have zero slope.
Vertical Lines(x=k) have undefined slope
Parallel lines have the same slope.
Perpendicular lines have opposite reciprocal slopes. $m = \frac{2}{3} \rightarrow m_{perp} = -\frac{3}{2}$

y-intercept is the y-coordinate of the point where the graph intersects the y-axis. The x-coordinate of this point is always 0. To find the y-intercept, replace x with 0 and solve for y.

x-intercept is the x-coordinate of the point where the graph intersects the x-axis. The y-coordinate of this point is always 0. To find the x-intercept, replace y with 0 and solve for x.

Variation: always involves the constant of variation *k*.

Direct Variation: $y = kx$

Inverse Variation: $y = \frac{k}{x}$

Joint Variation: $y = kxz$

Combined Variation: Sales "*y*" varies directly with advertising "*a*" and inversely with candy cost "*c*".
$y = \frac{ka}{c}$

Radicals: Use rational exponents

$\sqrt[a]{x} = x^{\frac{1}{a}}$

$\sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m = x^{\frac{m}{n}}$

$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

Simplify: look for perfect powers
$\sqrt{x^{12}y^{17}} = \sqrt{x^{12}y^{16}y} = x^6y^8\sqrt{y}$
$\sqrt[3]{72x^9y^8z^3} = \sqrt[3]{8 \cdot 9x^9y^6y^2z^3} = 2x^3y^2z\sqrt[3]{9y^2}$

Quadratic Equations: $ax^2 + bx + c = 0$
Set equation = 0. Solve by factoring, square root property, completing the square (CTS), and quadratic formula.

Square Root Property:
$If\ x^2 = m, \quad then\ x = \pm\sqrt{m}$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Discriminant: $b^2 - 4ac$. Used to determine number and type of solutions.

$b^2 - 4ac \begin{cases} > 0, & 2\ real\ unequal\ roots \\ = 0, & 1\ real\ root\ (repeated) \\ < 0, & no\ real\ solution \end{cases}$

Zero Product Property:
If ab=0, then a = 0 or b = 0

Solve by Factoring:
$x^2 + 2x = 8,$
$x^2 + 2x - 8 = 0, \quad set = 0$
$(x+4)(x-2) = 0, \quad Factor$
$x + 4 = 0\ or\ x - 2 = 0, Zero\ PP$
$x = -4\ or\ x = 2, solve\ each\ eq.$

Quadratic Functions

Standard Form: $f(x) = ax^2 + bx + c$

If a > 0, parabola opens upwards – function has a minimum.
If a < 0, parabola opens downwards-function has a maximum.
If $|a| > 1$, graph is narrower than y = x^2
If $|a| < 1$, graph is wider than y = x^2

Minimum or Maximum value given by the y-coordinate of the Vertex.

Axis Of Symmetry: $x = -\frac{b}{2a}$

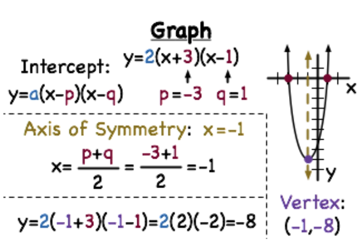
Vertex: $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

y-intercept: (0, c)

Quadratic Functions (Cont.)

Intercept Form:
$f(x) = a(x-p)(x-q)$
Where p and q are the x-intercepts (p,0) and (q, 0)

Axis of Symmetry is halfway between the intercepts: $x = \frac{p+q}{2}$
Plug it in to find the y-coordinate of the vertex. $V\left(\frac{p+q}{2}, f\left(\frac{p+q}{2}\right)\right)$



Completing the square:

$3x^2 - 6x - 1 = 0$	$3x^2 - 6x - 1 = 0$
1. Move constant to the right side.	$3x^2 - 6x = 1$
2. Rewrite the equation by factoring out the leading coefficient "a" from the quadratic and linear terms if ≠ 1.	$3(x^2 - 2x) = 1$
3. Rewrite with place holders.	$3(x^2 - 2x + \square) = 1 + 3\square$
4. Complete the Square (CTS)	$\left(\frac{b}{2}\right)^2 = \left(\frac{-2}{2}\right)^2 = (-1)^2 = 1$
5. Add the CTS value (Don't forget the leading coefficient)	$3(x^2 - 2x + 1) = 1 + 3(1)$
6. Factor and simplify.	$3(x-1)^2 = 4$
7. Divide by leading coefficient and solve using square roots (rationalize if necessary)	$(x-1)^2 = \frac{4}{3}, \quad x = 1 \pm \frac{2\sqrt{3}}{3}$

Vertex Form: $f(x) = a(x-h)^2 + k$

AOS: x = h

Vertex: (h, k)

y-intercept: Make x = 0

Inverses

***To find an inverse*.** Ex: $f(x) = x^3 + 4$

1. Rewrite f(x) as y = ; $y = x^3 + 4$

2. Switch x and y; $x = y^3 + 4$

3. Solve for y; $y = \sqrt[3]{x-4}$

4. Label appropriately

If a function $f^{-1}(x) = \sqrt[3]{x-4}$

Solving Systems of Linear Equations Methods:

Graphing
Substitution
Elimination

Ex: $\begin{cases} x + y = 4 \\ x - y = 6 \end{cases}$

Solution (5, -1)

Scatter Plots, Lines of Best Fit, Residuals, Correlation Coefficient r.

A **residual** is the vertical distance between a point and the function fitted to the data.

Residual = Actual y – Predicted y

A **residual plot** is a graph of the residuals (y-axis) versus the x-values (x-axis)

For a residual plot to be a good fit there should not be a pattern. It should be random.

The sum of the residuals is always 0.

The correlation coefficient r, measures the strength and direction of the linear association between two quantitative variables. r is unitless.

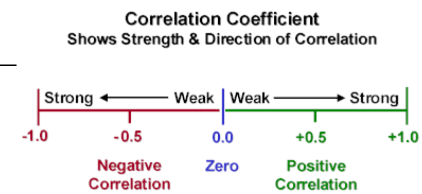
$-1 \le r \le 1$

The closer $|r|$ is to 1, the stronger is the relationship between x and y (points are closer to the line).

The closer $|r|$ is to 0, the weaker is the relationship between x and y.

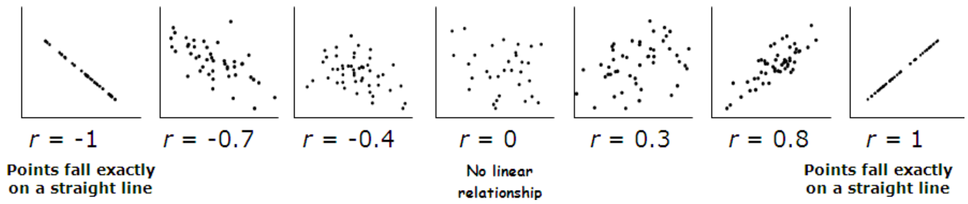
Correlation does not imply **causation.**

Causation is when one event causes another to happen. Ex. In the summer ice cream sales increase and shark attacks increase – they are correlated, but



Get the correlation coefficient (r) from your calculator or computer

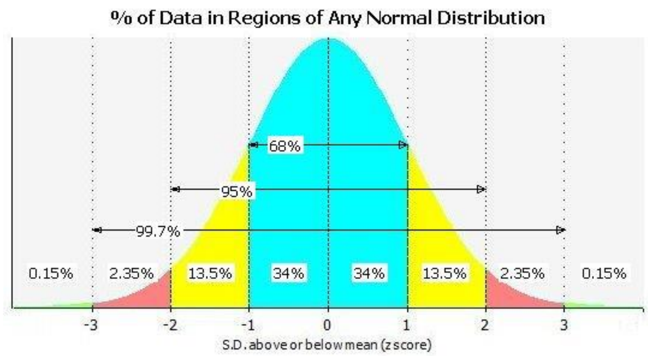
- r has a value between -1 and +1:



Vertical translation	$f(x) + k$ $k > 0$ up $k < 0$ down	$y = 2^x + 3$ *3 units up* $y = 2^x - 6$ *6 units down*
Horizontal translation	$f(x - h)$ $h > 0$ right $h < 0$ left	$y = 2^{x-2}$ *2 units right* $y = 2^{x+1}$ *1 unit left*
Reflection	$-f(x)$ across x-axis $f(-x)$ across y-axis	$y = -2^x$ across x-axis $y = 2^{-x}$ across y-axis
Vertical stretch or compression	$af(x)$ $a > 1$ stretch $0 < a < 1$ compression	$y = 6(2)^x$ *stretch by 6* $y = \frac{1}{3}(2)^x$ comp. by ⅓
Horizontal stretch or compression	$f\left(\frac{1}{b}x\right)$ $b > 1$ stretch $0 < b < 1$ compression	$y = 2^{\frac{1}{8}(x)}$ stretch by 8 $y = 2^{3x}$ comp by ⅓

Empirical Rule (or 68-95-99.7) Rule for Normal Distribution

- About 68% of all values fall within 1 standard deviation of the mean.
- About 95% of all values fall within 2 standard deviations of the mean.
- About 99.7% of all values fall within 3 standard deviations of the mean.



To identify if a function is linear, quadratic or exponential from a given set of data:
Linear: For every constant change in x, there is a constant **first** difference in y.
Quadratic: For every constant change in x, there is a constant **second** difference in y.
Exponential: For every constant change in x, there is a constant **ratio** in y.